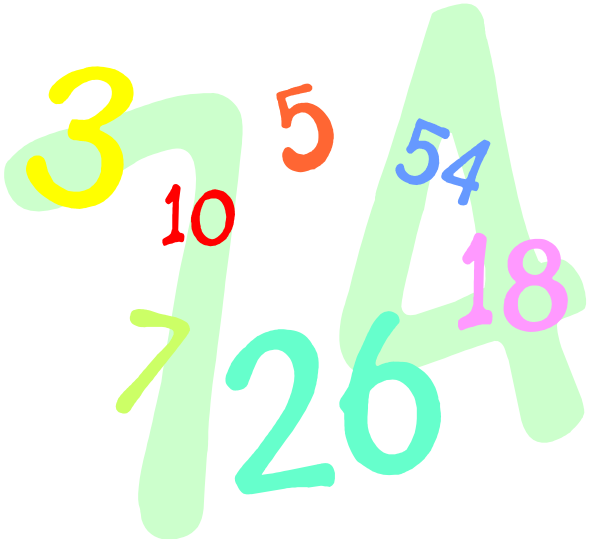
Calculation Policy

(Updated in August 2018)

Approved: Sept 2021, review date: Sept 2024

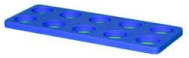
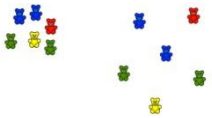


Introduction:

Children are introduced to the processes of calculation through practical, oral and mental activities. As they begin to understand the underlying ideas, they develop ways of recording to support their thinking and calculation methods, so that they develop both **conceptual understanding** and **fluency** in the fundamentals of mathematics. Whilst interpreting signs and symbols involved with calculation, orally in the first instance, children use models and images to support their mental and written methods of calculation. As children's mental methods are strengthened and refined they begin to work more efficiently, which will support them with using succinct written calculation strategies as they are developed.

From Early Years to Year 1:

There are fundamental skills that it is important for children to develop an early understanding of as building blocks to future learning in maths, including that linked to calculation. A selection of the skills include:

- Ordinality – 'the ordering of numbers in relation to one another' – e.g. (1, 2, 3, 4, 5…)
- Cardinality – 'understanding the value of different numbers' – e.g. (7 =  17 =  +  12 = 
- Equality – 'seven is the same total as four add three' – e.g.  = 
- Subitising – 'instantly recognizing the number of objects in a small group, without counting them' – e.g.  → five
- Conservation of number – 'recognising that a value of objects are the same, even if they are laid out differently' – e.g. 
- One-to-one correspondence – e.g. 

- Counting on and back from any number – e.g. 'five add three more totals eight'  'ten take away three totals seven' 

- Concept of zero  3 + 0 = 3
- Using apparatus and objects to represent and communicate thinking – e.g. 

The ability to calculate mentally forms the basis of all methods of calculation and has to be maintained. In the 2018 national Key Stage 1 SATs tests, every one of the named mental maths strategies below was assessed, whilst many also featured in a less explicit manner in the Key Stage 2 SATs tests, hence highlighting the need for each method to be taught explicitly. A good knowledge and 'feel' for numbers, is the product of structured practice through progression in relevant practical maths experiences alongside visual representations.

By the end of Year 6, children should be equipped with efficient mental and written calculation methods, which they use fluently. Decisions about when to progress should always be based on the security of pupils' understanding and their readiness to move ahead to the next stage. At whatever stage in their learning, and with whatever written method is being used, children's strategies must still be underpinned by a secure understanding and knowledge of number facts that can be recalled fluently with flexibility.

The overall aims are that when children leave primary school they:

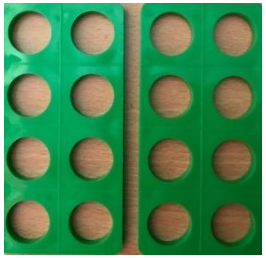
- Are able to recall number facts with fluency, having developed conceptual understanding through being able to visualise key ideas – such as those related to place value, through experience with practical equipment and visual representations;
- Make use of diagrams (including the bar model) and jottings to help record / reason through stages of thinking when using mental methods that generate more information than can be kept in their heads;
- Have an efficient, reliable, written method of calculation for each number operation that they can apply with confidence when undertaking calculations that they cannot carry out mentally;
- Are able to make connections between all four number operations, understanding how they relate to one another, as well as how the rules and laws of arithmetic can be applied.

.**Addition:**

Mental calculation strategies for addition and subtraction:

All mental calculation strategies need to be taught explicitly using a Concrete – Pictorial – Abstract (CPA) approach in every year group, for example, using decimals in Key Stage 2. The following ideas can be adjusted so that they are accessible to all children. The NCETM, 2015, state that, 'a pupil really understands a mathematical concept, idea or technique if he or she can represent it in a variety of ways.'

Doubles: 8 + 8 = 16



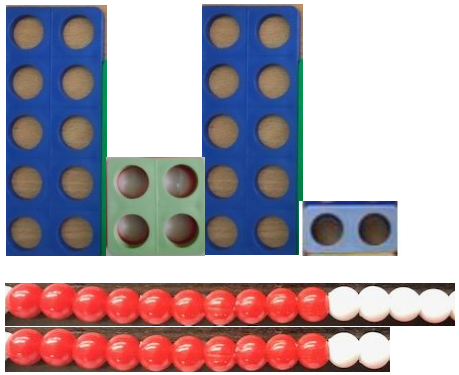
8 + 8 is connected to 8 X 2

Near doubles: 6 + 7 = 13

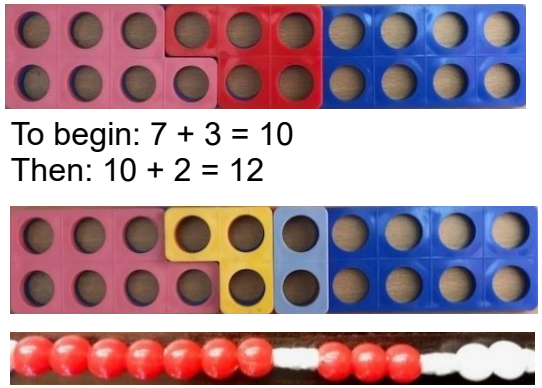
6 + 7 is commutative with 7 + 6

Number bonds: 7 + 3 = 10

Partitioning: 14 + 12 = 26



Bridging: 7 + 5 = 12



To begin: 7 + 3 = 10
Then: 10 + 2 = 12

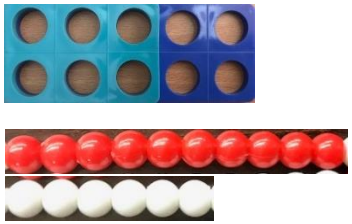
Adjusting: 16 + 9 = 25

To begin: 16 + 10 = 26

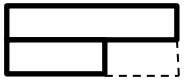
Then: 26 – 1 = 25

1	2	3	4	5	6	7	8	9	10
11	12	13	14	15	16	17	18	19	20
21	22	23	24	25	26	27	28	29	30
31	32	33	34	35	36	37	38	39	40
41	42	43	44	45	46	47	48	49	50

Finding the difference: 10 – 6 = 4



David has 10 sweets, whilst Chloe has six sweets. How many more does David have than Chloe?

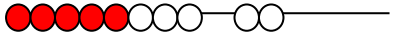


Reordering: 8 + 7 + 2 = 17
e.g. calculating numbers in a different order

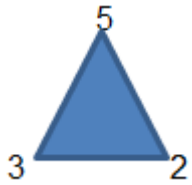
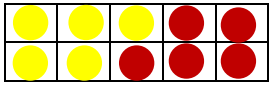
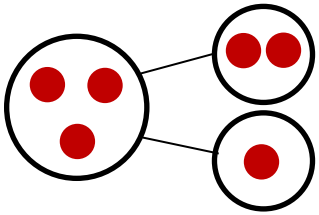
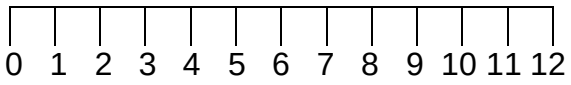
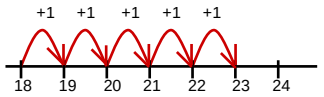
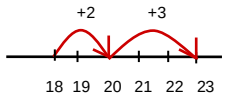
To begin: 8 + 2 = 10
Then: 10 + 7 = 17

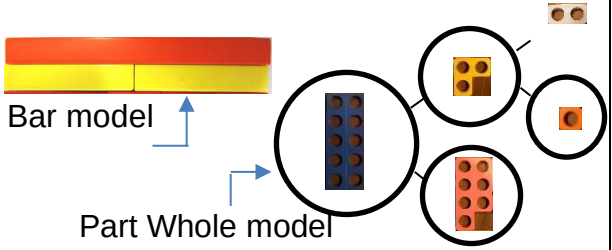
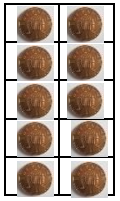
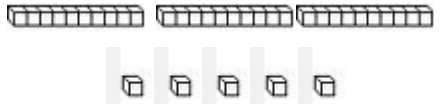
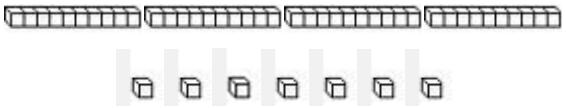
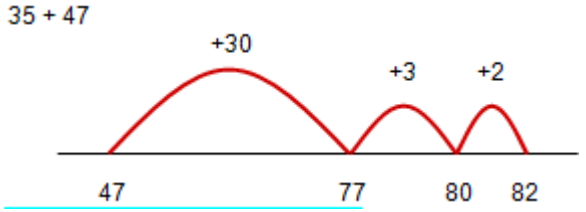
	Counting	**Mental maths strategies**	**Rapid recall**	**Written calculation and appropriate models and images to support conceptual understanding**
Stage 1:	Count in ones to and across 100 forwards and backwards starting from 0, 1 and other numbers.	<u>Explicitly teach every mental maths strategy detailed above.</u> Pupils use apparatus to explore addition as the inverse of subtraction.	Rapid recall of all pairs of numbers totalling numbers up to 20. Use structured apparatus –	**Combining two groups:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment.

3 + 2 = 5



‘eight add two more makes ten’

	Count in multiples of two, five and ten using a counting stick set up as a number track..	.   4 add 1 is 5 5 subtract 4 leaves 1	i.e. Numicon, tens frames, abaci, etc. 	• Teachers model use of number tracks to count on or line up counters/objects along the number track. This is a precursor to use of a fully numbered number-line. **Whole / part-whole model:** • The concept of a whole / part-whole model is introduced.	 'one more than four is five' 'Four add one more is the same as five'  Tens frame  Bar model  Part whole model
Stage 2:	Continue practicing above skills. Count in steps of 2, 3 and 5 forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as	Reorder numbers when adding, i.e. start with largest number, find bonds, etc. Add doubles and derive near doubles. Round numbers to the nearest 10.	Recall addition facts for all numbers to 20.	**Counting on from any number:** • Children begin to use numbered lines to support their own calculations, initially counting in ones before beginning to work more efficiently. **Counting on from the largest number:** • Children reorder calculations to start with the largest number. **Reordering calculations to apply use of mental maths strategies:**	Number line with all numbers labelled  18 + 5  …to… 

	well as a number square.			• Children reorder 'strings' of numbers to apply their understanding of mental maths strategies, including doubles and number bonds, e.g. 6 + 7 + 4 reordered to 6 + 4 = 10 and then 10 + 7 = 17. Jottings are used to help keep track of thinking. **Whole / part-whole model:** • The concept of a whole / part-whole model is reinforced and extended.	 Use of questions such as: 'How might I rearrange these to find the total?' 
Stage 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on by 10 or 100 from any two digit number. Link to counting stick: counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	Partitioning by bridging through 10 and multiples of 10 when adding. Adjusting when adding 11 or 9 to a number. Relating inverse number operations – using structured apparatus to explore and understand that subtraction undoes addition.	Connect pairs totalling ten to pairs of multiples of 10 totalling 100.  Use 10ps in tens frame. Recall pairs of two-digit numbers with a total of 100, i.e. 32 + ? = 100.	**Expanded horizontal addition:** • Add numbers using structured apparatus to support understanding of place value. • Make connections between partitioning both numbers using structured apparatus and partition the second number only using a number line.	Add…  …and…  By partitioning and recombining 30+ 40 = 70 5 + 7 = 12 70 + 12 = 82 

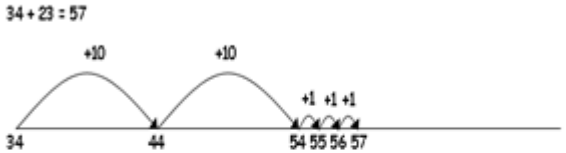
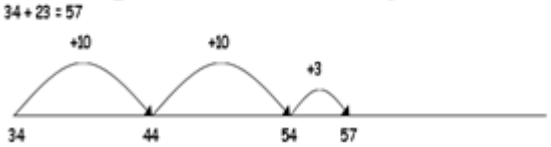
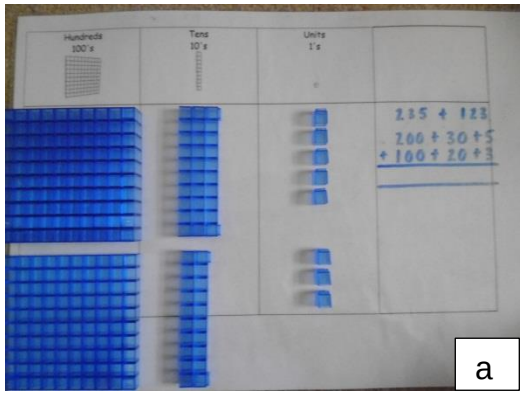
Stage 4:	Continue practicing previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc. Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes. Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 8 + 3 = 11, I also know 0.8 + 0.3 = 1.1 and 8/100 + 3/100 = 11/100.' Sums and differences of pairs of multiples of 10, 100 or 1000. Addition doubles of numbers to 100. Pairs of fractions totalling 1.	**Expanded horizontal method, leading to columnar addition:** • Written recording should follow teacher modelling around the size of numbers and place value using a variety of concrete materials, e.g. straws, Numicon, Dienes and place-value cards. • Teachers model how numbers can be partitioned into tens and ones, as well as in different ways, e.g. 20 + 5 10 + 15 • As children move towards using a columnar method, links continue to be made with earlier models and images, including the number line.	It is crucial that empty number lines are kept as well as using more formal written calculation methods. Counting on in tens and ones to solve an addition calculation:  Counting on more efficiently: 

Illustration of how to use Dienes equipment to ensure children have an understanding of place value when using columnar addition.



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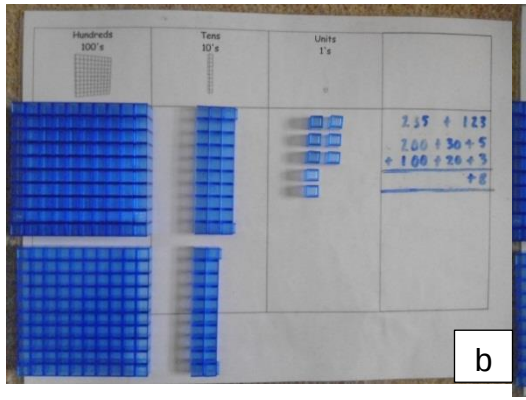

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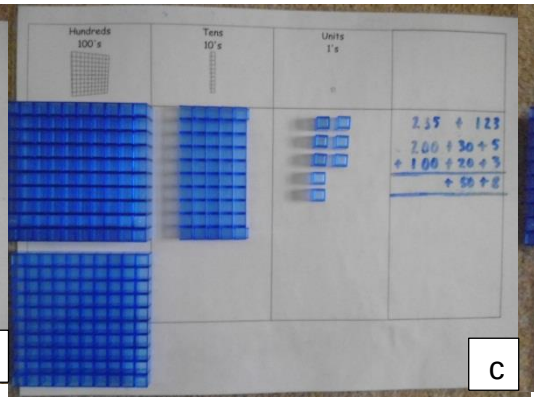

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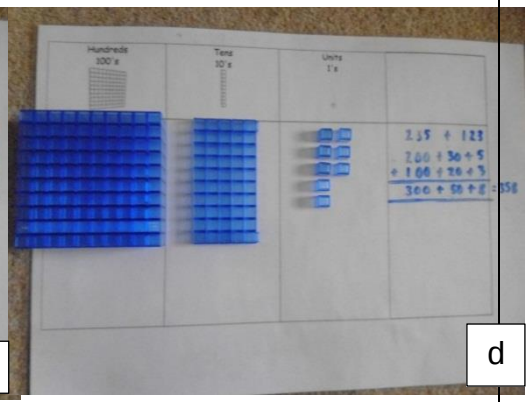

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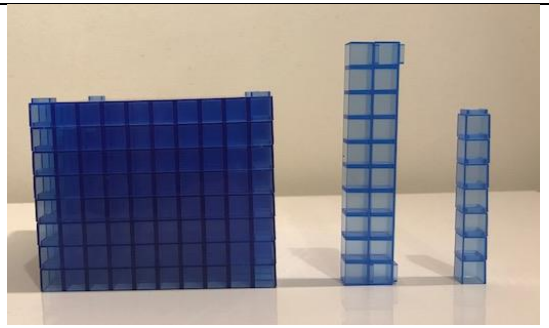
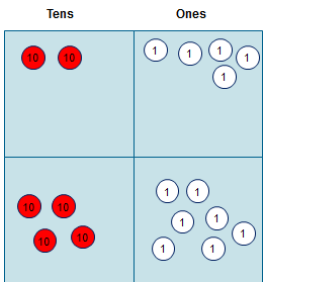
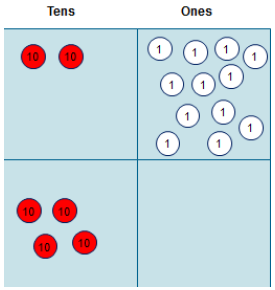
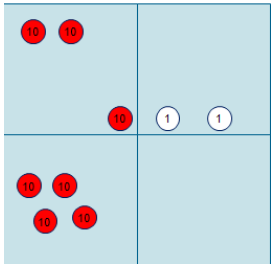
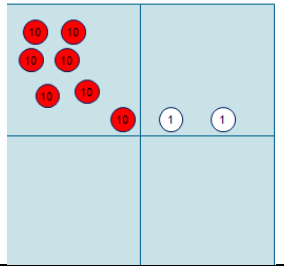
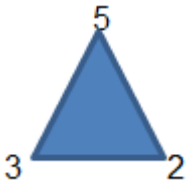
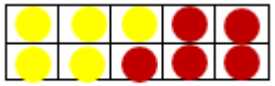
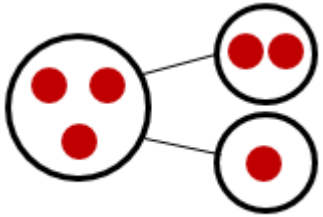
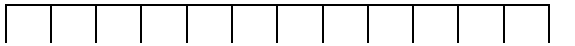
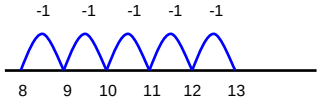
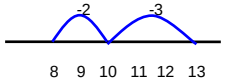
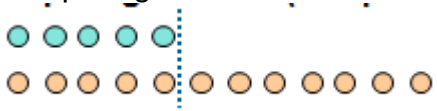
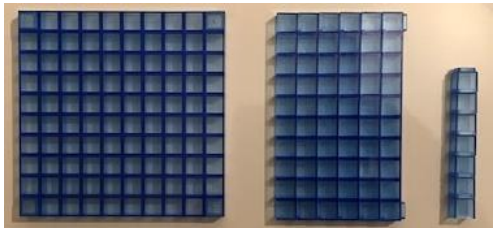
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to add decimals, i.e. 3.4 + 2.5 = 5 + 0.9  Reorder increasingly complex calculations, i.e. 1.7 + 2.8 + 0.3 = 1.7 + 0.3 + 2.8 Compensating – i.e. 405 + 399 → add 400 and then subtract 1.	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages and decimals Doubles and halves of decimals, i.e. half of 5.6, double 3.4. Sums and differences of decimals, i.e. 6.5 + 2.7	**Expanded vertical method, leading to columnar addition:** • Teachers model a column method that records and explains partial mental methods. • There remains an emphasis on the language of calculation, e.g. 'Forty plus seventy equals one-hundred and ten.'… 'Seven add six equals thirteen.' …before recombining numbers. Teachers also model the language of: 'Four tens add seven tens total eleven tens or 110.' • Teachers similarly advance to model the addition of two 3-digit numbers with the expectation that as children's knowledge of place value is secured, they become ready to approach a formal compact method.	Informal columnar: Adding the tens first: 76 +47 ___ 110 13 ____ 123 _____ Adding the hundreds first: 471 + 356 = 700 + 120 + 7 = 827 Adding the ones first: 471 + 356 = 7 + 120 + 700 = 827

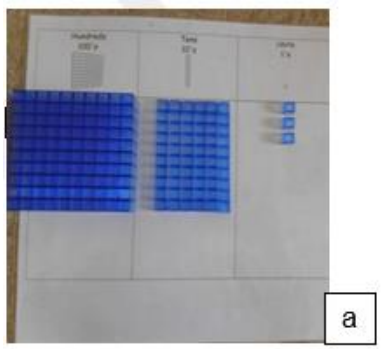
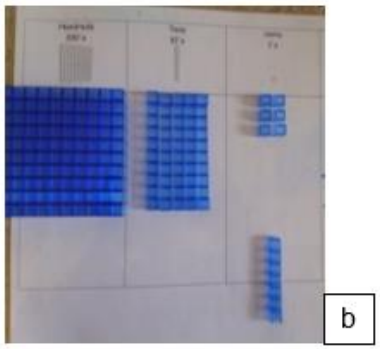
					
Stage 6:	Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 0.8 + 0.35 = 0.8 + 0.2 + 0.15 using empty number lines. Partitioning using near doubles, i.e. 2.5 + 2.6 = 5 + 0.1 Reorder decimals, i.e. 4.7 + 5.6 – 0.7 …as… 4.7 – 0.7 + 5.6 = 4 + 5.6.	Ensure all children are confident recalling basic facts to 20 and deriving facts using place value. Make links between decimals, fractions and percentages.	**Columnar addition (formal written method):** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have two tens and five ones, which need adding to four tens and seven ones." 2. "I add five ones to seven ones, which gives me twelve ones." 3. "I exchange ten of my twelve ones for a ten counter." 4. "I add my three tens and four tens to make seven tens." "Altogether, I have seven tens and two ones." • Teachers similarly advance to model the addition of two 3-digit numbers, e.g.	**Pupils to be encouraged to consider mental strategies first.** Formal columnar: 25 +47  25 +47 2 1   25 +47

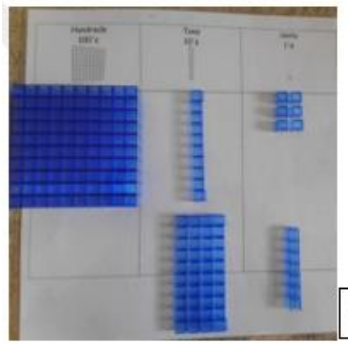
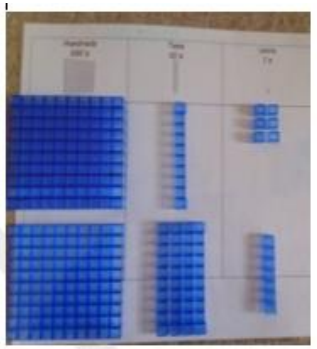
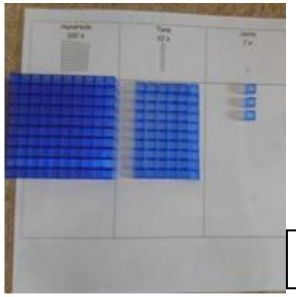
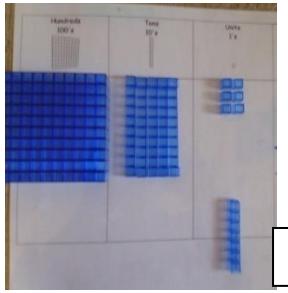
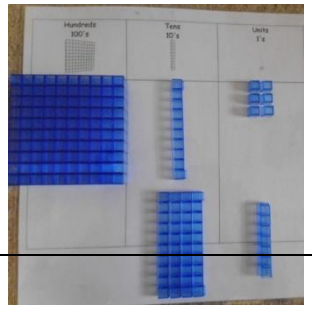
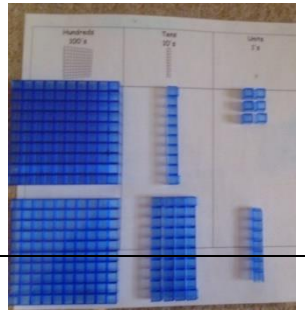
				587 + 475 1062 1 1	2 1 Tens Ones  25 +47 72 1 Tens Ones 

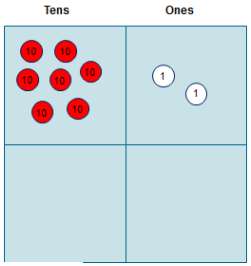
Subtraction:

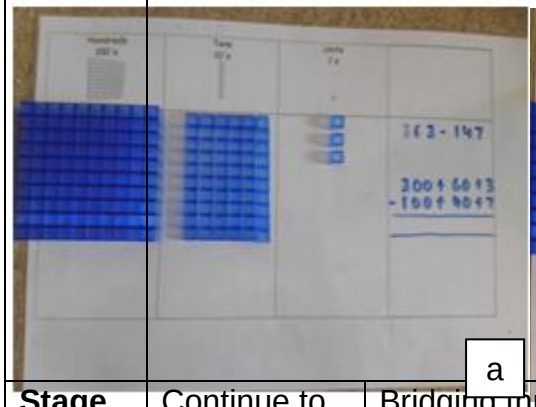
	Counting	Mental strategies	Rapid Recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count in ones to and across 100, forwards and backwards starting from 0, 1 and other numbers. Count in multiples of two, five and ten.	Pupils use apparatus to explore addition as the inverse of subtraction: .   'four add one is five.' 'five subtract four leaves one'	Rapid recall of subtraction facts for numbers up to 10. Use structured apparatus, i.e. Numicon, tens frames, abaci etc. 	**Subtraction as taking away from a group:** • Children develop a mental picture of the number system for use with calculation. A range of key models and images support this, alongside practical equipment. • Teachers model use of number tracks to count back or remove counters/objects from the number track or set. This is a precursor to use of a fully numbered number-line.	  5 - 2 = 3  'six take away two leaves four'  'one less than six is five'  Tens frame  Bar model  Cherry model
Stage 2:	Continue practicing above skills. Count in steps of 2, 3	Bridging through two digit numbers, i.e. 24 – 19 = 19 + 1 + 4 using number lines.	Recall subtraction (and addition) facts for all	**Subtracting by counting back and on:** • Children begin to use	Number line with all numbers labelled 

	and 5, forwards and backwards to and from zero. Count in tens from any number – link to coins in a piggy bank as well as a number square.	Subtracting 11 by subtracting 10 and then 1 more. Move to subtracting 9 by subtracting 10 and adding 1 using apparatus.	numbers to 20.	numbered lines to support their own calculations, initially counting back in ones before beginning to work more efficiently. **Finding the difference:** • Teachers model how to find the difference when two numbers are relatively 'close together.'	13 – 5 = 8  13 – 5 = 8  Counting back only to be developed as a mental strategy where it is sensible to do so 51-3= 125-32 etc Comparing two sets to find the difference. 
Stage 3:	Continue practicing above skills. Count from 0 in multiples of 4, 8, 50 and 100. Count on and back by 10 or 100 from any two digit number. Link to counting	Partitioning by bridging through 10 and multiples of 10 when subtracting. Continue to practice adjusting when subtracting 11 or 9 from a number. Relating inverse number operations – use structured apparatus to explore and	Connect subtractions from ten to subtractions from multiples of 10 totalling 100. 	**Taking away:** • When teaching children about reduction, highlight the importance of only partitioning one number.	Subtraction by partitioning with use of manipulatives and linked with a horizontal expanded written algorithm: 167 – 24 = 143 20 4 

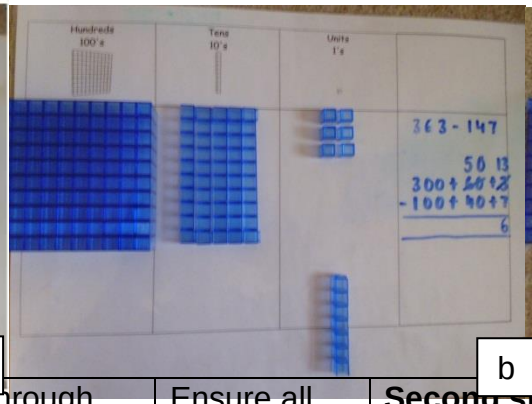
	stick counting forwards and backwards flexibly. Count up and down in tenths – linking to visual image.	understand that subtraction undoes addition.	Use 10ps in tens frame. Subtract two digit numbers from 100 i.e. ? = 100 - 78		In either order… To begin: 167 – 20 = 147 Then: 147 – 4 = 143  100 + 60 + 7 - 20 + 4 0 + 40 + 3 Finding the difference on a number line:  Children should note that finding the difference is often the most efficient way of solving a subtraction problem when two numbers are close together. e.g. 61 – 59
Stage 4:	Continue practicing of previous skills. Count forwards and backwards from 0 in multiples of 6, 7, 9, 25 and 1000 using counting sticks, number lines, number squares, etc.	Bridging through 60 for time, i.e. 70 minutes = 1 hour and 10 minutes Rounding any number to the nearest 10, 100 or 1000. Rounding numbers with one decimal place to nearest whole number. Explore inverse as a way to derive new facts and to check accuracy of answers.	As above. Use known facts and place value to derive new ones, i.e. 'If I know 11 - 3 = 8, I also know 1.1 - 0.3 = 0.8 and 8/100 - 3/100 = 5/100.' Sums and differences of pairs of multiples of	**Taking away:** • Subtraction by partitioning with use of manipulatives, and including transfer / exchange, linked with a horizontal expanded written algorithm in preparation for a future formal column method.	363 – 147 = 216 50 13 300 + ~~60~~ + ~~3~~ - 100 + 40 + 7 200 + 10 + 6  

	Count up and down in tenths, hundredths and simple fractions using models and images, i.e. Dienes equipment, counting stick, ITPs.		10, 100 or 1000. Subtraction of fractions totalling 1, i.e. 1 – 0.3 = 0.7		 c  d 
Stage 5:	Count forwards and backwards in steps of powers of 10 for any given number up to one million. Continue to count forwards and backwards in simple fractions. Count forward and backwards in appropriate decimals and percentages.	Use apparatus and knowledge of place value to subtract decimals, i.e. 3.8 - 2.5 = 1.3 Reorder increasingly complex calculations, i.e. 1.7 – 5 – 0.7 = 1.7 – 0.7 – 5. Compensating – i.e. 405 - 399 → subtract 400 and then add 1.	Continue to practice previous stage and make links between known facts and addition pairs for fractions, percentages and decimals. Doubles and halves of decimals, i.e. half of 5.6, double 3.4.	**Column method with Dienes:** Subtraction by partitioning with use of manipulatives, and including transfer / exchange, linked with a formal column written algorithm.	5 1 3 ~~6~~3 - 1 4 7 2 1 6  a  b  

			Sums and differences of decimals, i.e. 6.5 + 2.7		
	363 – 147 = 216 50 13 300 + 60 + 3 100 + 40 + 7 200 + 10 + 6 = 216	Illustration of how to use Dienes equipment to ensure children understand transference of numbers when using columnar subtraction.			
Stage 6:	Continue to practice previous skills. Count forwards and backwards in simple fractions, decimals and percentages.	Bridging through decimals, i.e. 1.5 – 0.8 = 1.5 – 0.5 then -0.3 using empty number line. d	Ensure all children are confident recalling basic facts to 20 and deriving using place value. Make links between decimals, fractions and percentages.	**Second stage of column method:** • The concept of exchange is introduced through continued use of practical equipment (manipulatives). • Teachers model: 1. "I have seven tens and two ones. I need to subtract	Formal columnar: 7 2 - 4 7  6 1 7 2 - 4 7



a



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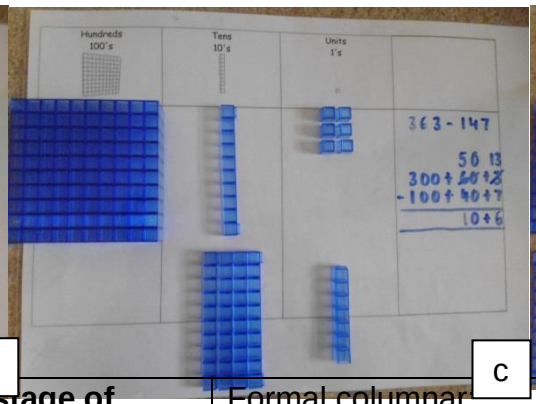

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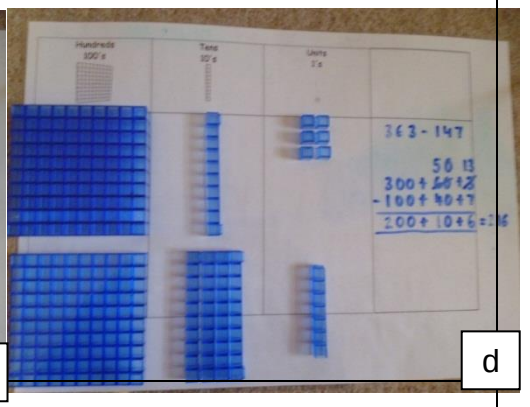
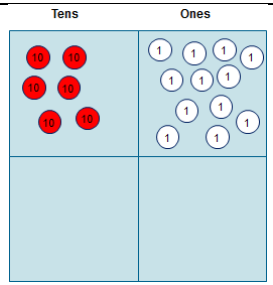
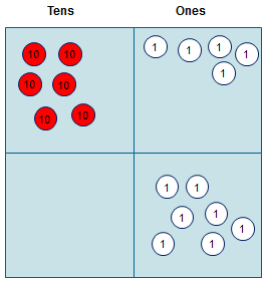
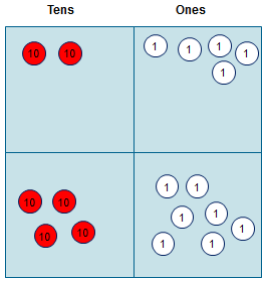


d

				four tens and seven ones." 2. "At the moment, I cannot subtract seven ones from two ones, so I need to transfer one ten to become ten ones." 3. "Now I can take away seven ones from twelve ones, so that I have fives ones left. 4. "I can now subtract four tens from six tens, which leaves me with two tens." 5. "I recombine two tens and fives ones to understand that I am left with twenty-five."	    

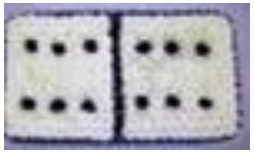
				• Teachers similarly advance to model the subtraction of one 3-digit number from another, e.g. 5 1 5 ~~6~~ 3 2 4 6 3 1 7	

Multiplication:

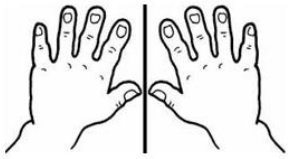
Mental calculation strategies for multiplication and division:

Doubling and halving:

Double six is 12…



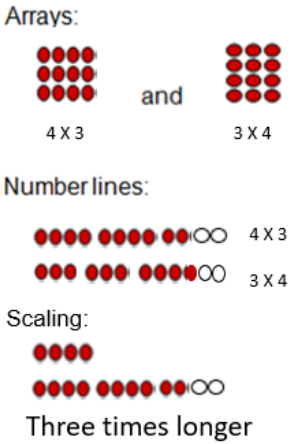
Double five is ten…



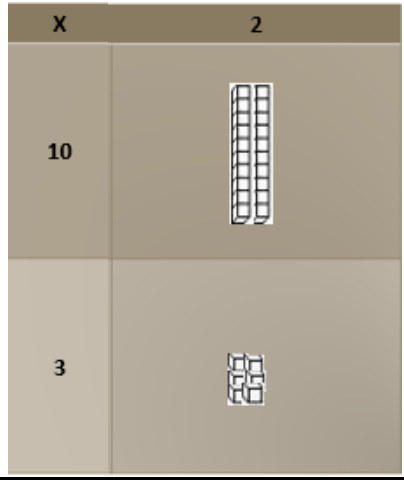
Double 16 can be calculated by working out…
Double ten → 20
Double six → 12

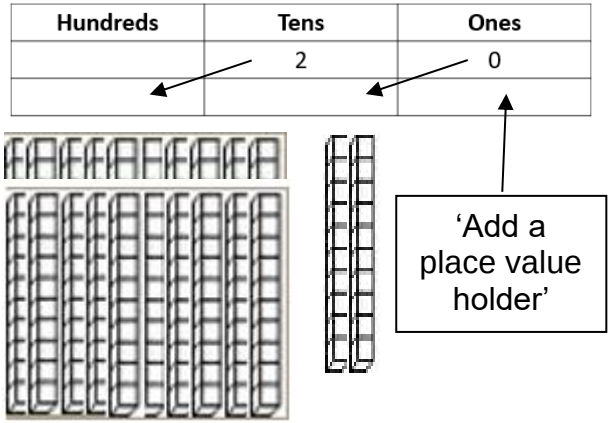
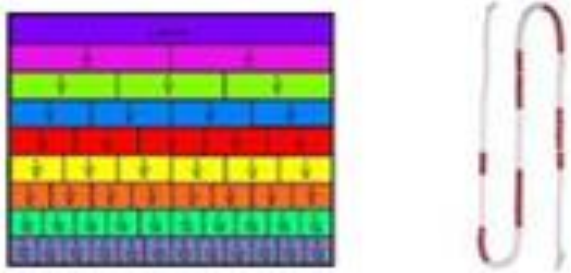
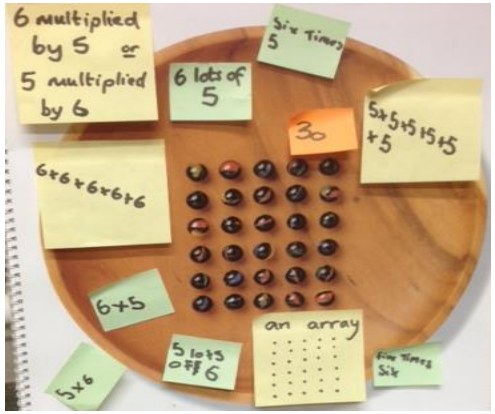
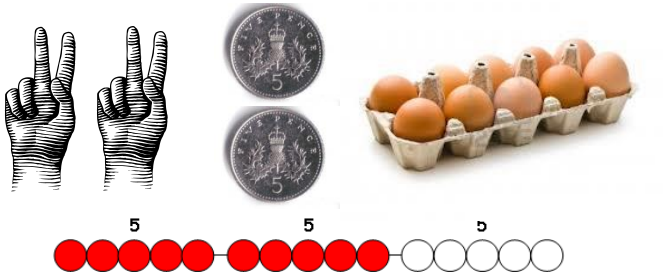
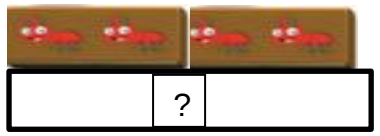
With links to finding four-times a given value and finding a quarter of a value.

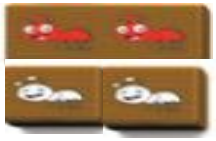
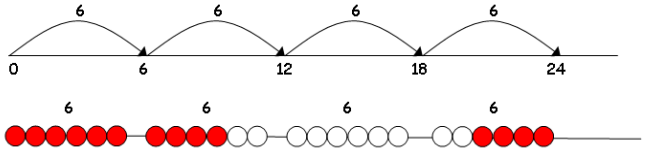
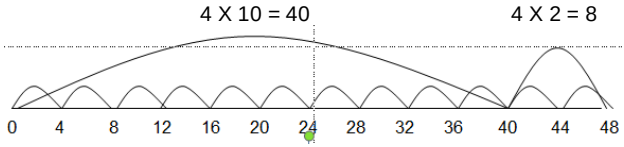
Knowing multiplication and division facts to 12 X 12:

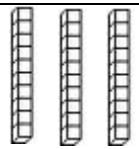
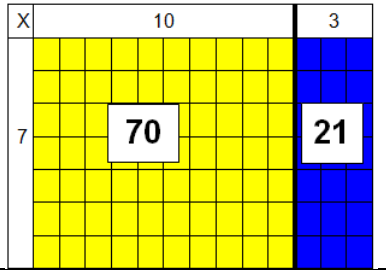
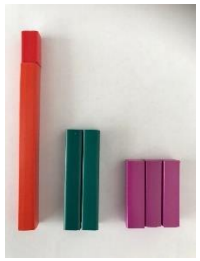


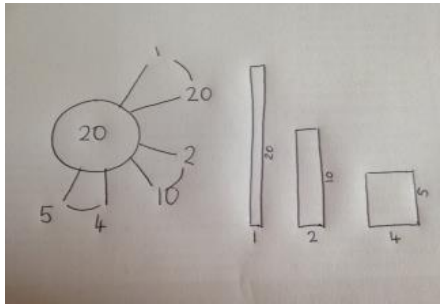
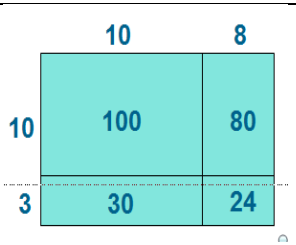
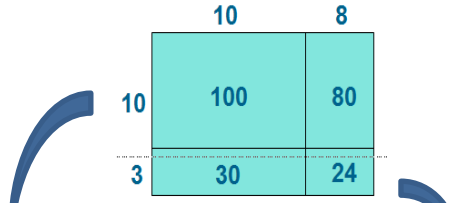
Multiplying a teen number by one-digit number:



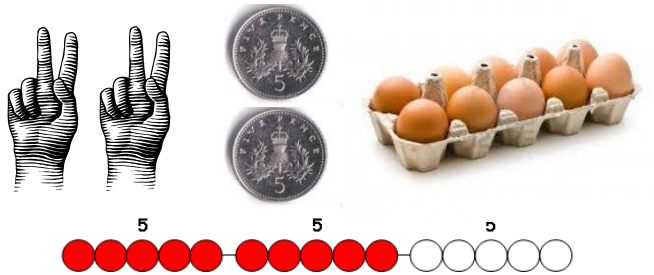
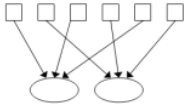
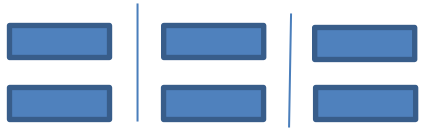
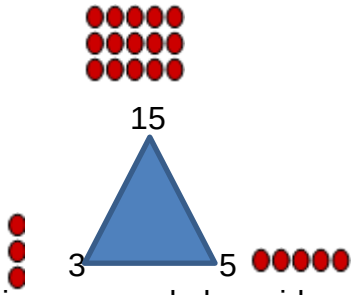
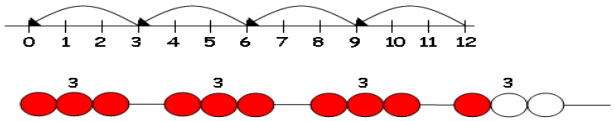
	Multiplying and dividing by multiples of ten: 20 X 10 = 200 		**Identifying fractions, decimals and percentages:** 		Milk the maths…  ...by allowing children to make connections between number facts.	
	Counting	**Mental strategies**	**Rapid recall**	**Written calculation and appropriate models and images to support conceptual understanding**	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten. Recall odd and even numbers to 10 in reference to structured apparatus. 	**Developing early conceptual understanding of multiplication:**	Use objects, pictorial representations and arrays to show the concept of multiplication:  Early bar model 

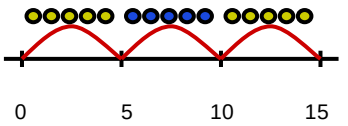
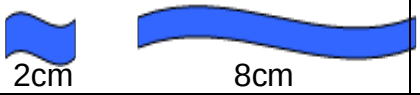
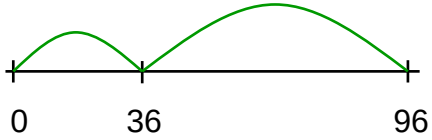
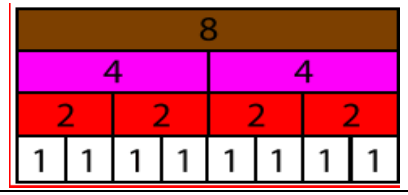
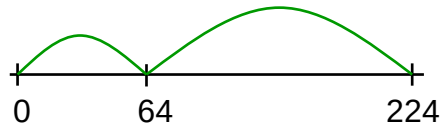
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations: 10 2 5 Stories are used alongside a triad to help children understand links between number operations, e.g. “There are five pencils in two packs, which means that there are ten pencils altogether.” Doubling is reinforced using a whole/part-whole model: 	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to structured apparatus. Recall & use multiplication facts for the 2X, 5X and 10X-tables.	**Understanding multiplication as repeated addition:** • Investigate multiplication as repeated addition, so that the law of cummutativity is understood. • Whilst arrays are also modelled explicitly at this stage, it is important to note that they will continue to be a key model at later stages, alongside more formal methods of calculation.	Arrays: 5 X 3  and 3 X 5  Connect related facts with both array and repeated addition images. Number lines: 6 X 4 = 24  So: ‘Six multiplied by four’ …or… ‘Six taken four times.’
Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4	Recall odd and even numbers to 100 in reference to structured apparatus.	**Relate multiplying a 2-digit by 1-digit number using repeated addition and arrays to represent:**	Children use an empty number line to chunk efficiently: 4 X 12 = 48  3 X 13 = 39

	Count up and down in tenths.	Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon  2cm 8cm	Recall and use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.		<table><tr><td>X</td><td>10</td><td>3</td></tr><tr><td>3</td><td>  </td><td>  </td></tr></table> 7 X 13 = 91 
Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero. Count up and down in tenths and hundredths.	Derive factor pairs of numbers using models and images, e.g.  Use reordering to multiply three numbers. Children learn about the associative law: (9 X 5) X 10 = (10 X 5) X 9	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 3/2-digit by 1-digit number with arrays towards using long/short multiplication:**	Relate multiplying a 3/2-digit by 1-digit number, now also setting it out as short multiplication. 114 X 2 = 228 <table><tr><td>X</td><td>100</td><td>10</td><td>4</td></tr><tr><td>2</td><td>  </td><td>  </td><td>  </td></tr></table> 114 X 2 = 100 X 2 = 200 10 X 2 = 20 4 X 2 = 8 = 228 Link with distributive law: (100 X 2) + (10 X 2) + (4 X 2) = 228 At this stage, the **non-statutory** guidance in the national curriculum suggests teaching short

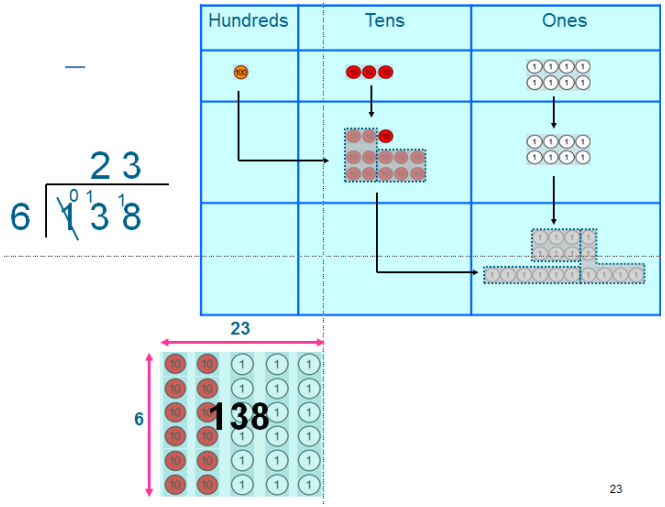
		Know what happens when a number is multiplied by zero or one.			multiplication; however, the team feel that an expanded form of calculation (as set out above) is be a better lead into long/short multiplication.
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers. 	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using long multiplication:**	 18 X13 24 30 80 100 234
Stage 6:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and operations.	Recall & use multiplication facts for all times-tables up to 12 X 12. In addition, use facts confidently to make larger calculations.	**Relate multiplying a 4/3/2-digit by 1/2-digit number with grid to using short multiplication:**	 18 X13 180 54 234 Once children have fully grasped the concept of multiplication alongside manipulatives and an expanded written method, they will be well-placed to progress towards a more compact written algorithm.

Division:

	Counting	Mental strategies	Rapid recall	Written calculation and appropriate models and images to support conceptual understanding	
Stage 1:	Count forwards and backwards in 2s, 5s and 10s	Doubling up to six and then ten whilst using related models and images.	Derive/recall doubles up to five and derive/recall halves up to ten. Recall odd and even numbers to 10 in reference to structured apparatus. 	**Developing early conceptual understanding of division as grouping and sharing:**	Use objects, pictorial representations and arrays to show the concept of division as grouping and sharing.  "Two children share six pencils between them"  "Six children are asked to get into three equal groups" 
Stage 2:	Count forwards and backwards in 2s, 3s, 5s and 10s from zero.	Begin to understand and use inverse number operations.  Stories are used alongside a triad to help children	Derive/recall doubles up to ten and derive/recall halves up to twenty. Recall odd and even numbers to 20 in reference to	**Understanding division as repeated subtraction:** • Investigate division as repeated subtraction. • Through teacher modelling, children need	Number lines and arrays: 12 ÷ 3 = 4 

		understand links between number operations, e.g. “15 children are asked to get into three groups and find out that there are five people in each group.”	structured apparatus. Recall and use multiplication facts for the 2X, 5X and 10X-tables.	to know that division is not commutative.	15 ÷ 5 = 3  Early bar model  Emphasis should be placed on solving division as grouping not sharing at this stage.
Stage 3:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 8s and 10s from zero.	Use doubling to make connections between the 2X, 4X and 8X-tables. Understand that multiplication can be undertaken by partitioning numbers, e.g. 12 X 4 = 10 X 4 + 2 X 4 Introduce the structure of scaling: e.g. Find a ribbon that is 4 times as long as the blue ribbon.  	Recall odd and even numbers to 100 in reference to structured apparatus. Recall & use multiplication facts for the 2X, 3X, 4X, 5X, 8X and 10X tables.	**Dividing a 2-digit by 1-digit number, representing this efficiently on a number line:**	Children use an empty number line to chunk efficiently. 96 ÷ 6 = 16 6 x 6 = 36 10 x 6 = 60  Conceptual understanding can be provided through use of a bead string to highlight the chunks. 
Stage 4:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 7s, 8s, 10s, 25s and 1000s from zero.	Derive factor pairs of numbers using models and images e.g. Cuisenaire.  	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 3/2-digit by 1-digit number, representing this efficiently on a number line, also in relation to long division:** • At this stage, no	Children use an empty number line to chunk efficiently. 224 ÷ 8 = 28 8 x 8 = 64 20 x 8 = 160 

		Know what happens when a number is multiplied by zero or one. Use reordering to multiply three numbers.		remainders are present unless in a practical context.	28 8 | 224 - 160 (8 X 20) 64 - 64 (8 X 8) 0 …or… 28 8 | 224 20 X 8 = 160 64 8 X 8 = 64 0
Stage 5:	Counting forwards and backwards in 2s, 3s, 4s, 5s, 6s, 7s, 8s, 9s, 10s, 25s and 1000s from zero.	Identify multiples and factors, including finding all factor pairs of a number, and common factors of two numbers.	Recall & use multiplication facts for all times-tables up to 12 X 12.	**Dividing a 4/3/2-digit by 1-digit number, in relation to long division:** • By this stage, there is a statutory requirement that children can use a formal written calculation method, such as long division. • Short division may begin to be taught alongside long division, but still with use of visual representations	Remainders should be interpreted in the following ways when long division is used: • as whole numbers • as fractions • through rounding in an appropriate way to the context Long division: 415 ÷ 9 = 46 and 1/9 46 and 1/9 9 | 415 - 360 (9 X 40) 55 - 54 (9 X 6) 1
Stage 6:	Consolidate all previous counting, including forwards and	Perform mental calculations, including with mixed numbers and different number operations.	Recall & use multiplication facts for all times-tables up to 12 X 12. In	**Dividing a 4/3/2-digit by 2/1-digit number, in relation to long and then short division:**	.Remainders should be interpreted in the following way when short division is used: • through rounding in an appropriate way to the context Long division: 432 ÷ 15 = 28 4/5

	backwards in fractions.		addition, use facts confidently to make larger calculations.	• By this stage, there is a statutory requirement that children can use formal written calculation methods, including long and short division. • Use of visual representations – like the ones opposite – remain important.	28 15 | 432 20 X 15 = 300 132 8 X 15 = 120 12 $\frac{12}{15} = \frac{4}{5}$ Answer: $28\frac{4}{5}$ Short division: 138 ÷ 6 = 23  Key language: 'How many groups of six one-hundreds are there in one-hundred?' 'How many groups of six tens are there in thirteen tens?' 'How many groups of six ones are there in eighteen?'

Stage 7:	Consolidate all previous counting, including forwards and backwards in fractions.	Perform mental calculations, including with mixed numbers and different number operations.	Tables facts to 12 x 12 Multiply by 10, 100 and 1000 with ease. Double and halve 2-digit numbers	**Short division for decimals and converting fractions to decimals.**	Extend the children's understanding of short division to decimals, using their understanding of place value. e.g. 9.8 ÷ 7 = $\begin{array}{r} 1.4 \\ 7\overline{)9.^{2}8} \end{array}$ e.g. Write 3/8 as a decimal $\begin{array}{r} 0.375 \\ 8\overline{)3.^{3}0^{6}0^{4}0} \end{array}$ Make sure that they do not attempt to combine a decimal division with a fraction remainder.